F-CONTINUOUS FUNCTIONS AND SUB-F-CONTINUOUS
FUNCTIONS
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Abstract:
In this paper we introduce and study F-closed sets and new types of generalized
continuity.

Introduction:
A subset A of a topological space X is said to be F-closed if it is the intersection of an
open and closed set. In this paper we introduce three different notions of generalized
continuity, namely F- irresoluteness, F-continuity and sub-F- continuity and we
discuss some properties of these functions.

Definition (1-1):
A subset A of a space (X, τ) is called F-closed if A=U∩V such that U is open set and
V is closed set in X. We denote the collection of all F-closed subsets of X by F(X, τ).

Remarks (1-2):
A subset A of X is F-closed set iff X-A is the union of an open set and a closed set.
1. Any open (resp. closed) subset of X is F-closed set.
2. The complement of a F-closed subset need not be F-closed set.

Definition (1-3):
A subset A of a space (X, τ) is said to be preopen set if A⊆ int(cl A).

Remarks (1-4):
1. Every open set is preopen set.
2. Every preopen and F-closed set is open set.
Proposition (1-5):
Let $A$ be a subset of a space $(X, \tau)$, then the following statements are equivalent:

1. $A$ is $F$-closed set.
2. $A = U \cap \text{cl } A$, $U$ is open set in $X$.
3. $\text{cl } A - A$ is closed set.

Remark (1-6):
Let $A$ any sub set of a space $(X, \tau)$ then $A$ need not be $F$-closed set, but if $(X, \tau)$ has property which every dense subset of $X$ is open set then $A$ is $F$-closed set.

Proposition (1-7):
Let $A$ and $B$ be $F$-closed subsets of a space $(X, \tau)$. If $A \cap \text{cl } B = \text{cl } A \cap B = \emptyset$, then $A \cup B \in F(X, \tau)$.

Proof:
Suppose there are open sets $U$ and $V$ such that $A = U \cap \text{cl } A$ and $B = V \cap \text{cl } B$.
Since $A \cap \text{cl } B = B \cap \text{cl } A = \emptyset$, then $A \cup B = (U \cup V) \cap \text{cl } (A \cup B)$, from definition of $F$-closed set we obtain $A \cup B \in F(X, \tau)$.

Definition (1-8):
A function $f : (X, \tau) \to (Y, \tau')$ is said to be $F$-irresolute function iff for any $F$-closed set $U$ in $Y$ then $f^{-1}(U)$ is $F$-closed set in $X$.

Definition (1-9):
A function $f : (X, \tau) \to (Y, \tau')$ is said to be $F$-continuous function iff for any open set $U$ in $Y$ then $f^{-1}(U)$ is $F$-closed set in $X$.

Definition (1-10):
A function $(X, \tau) \to (Y, \tau')$ is said to be sub-$F$-continuous function if there is a subbase or base $B$ for $Y$ such that for any $U \in B$ then $f^{-1}(U)$ is $F$-closed set in $X$. 
Theorem (1-11):

Let \( f : (X, \tau) \rightarrow (Y, \tau') \) be a function, then

1. If \( f \) is continuous function then \( f \) is F-irresolute function.
2. If \( f \) is F-irresolute function then \( f \) is F-continuous function.
3. If \( f \) is F-continuous function then \( f \) is sub-F-continuous function.

Remark (1-12):

The converse of theorem above is not true in general. The following examples explain that.

Example (1-13):

Let \( f : (R, \tau_U) \rightarrow (R, \tau_U) \), \( \tau_U \) is usual topology on \( R \), we will define \( f \) on \( R \) as follows:

\[ f(x) = 1 \text{ if } x > 0 \quad \text{and} \quad f(x) = x \text{ if } x \leq 0 \]

We note that \( f \) is not continuous function but \( f \) is F-irresolute function because for any F-closed set \( U \) in \( R \) then \( f^{-1}(U) = U \cup (0, \infty) \) if \( 1 \in U \) and \( f^{-1}(U) = U \cap (-\infty, 0) \) if \( 1 \notin U \), \( U \cup (0, \infty) \) and \( U \cap (-\infty, 0) \) are F-closed sets, therefore, \( f \) is F-irresolute function.

Example (1-14):

Let \( f : (R, \tau_U) \rightarrow (R, \tau_U) \) such that \( f(x) = x \) if \( x \neq 0 \) and \( f(0) = 1 \). For any \( U \subset R \) we have \( f^{-1}(U) = U \cap \{0\} \) if \( 1 \notin U \) and \( f^{-1}(U) = U \cup \{0\} \) if \( 1 \in U \).

Hence, if \( U \) is an open interval then \( f^{-1}(U) \) is F-closed. Thus \( f \) is sub-F-continuous function, but \( f \) is not F-continuous function because there is an open set \( U = R \setminus \{0\} \cup \{1 \mid n \in \mathbb{N} \text{ and } n \geq 2 \} \) and \( f^{-1}(U) = \{x \in \mathbb{R} \mid x \neq 1 \text{ for each } n \geq 2\} \) is not F-closed set.

Example (1-15):
Let $E=\{1^n \mid n \in \mathbb{N}\}$, let $f : (\mathbb{R}, \tau_u) \rightarrow (\mathbb{R}, \tau_u)$ such that $f(x)=x$ if $x \in E$ and $f(x)=0$ if $x \in \mathbb{R} \setminus E$, $f$ is not F-irresolute function because $\{0\}$ is F-closed set in $\mathbb{R}$ but $f^{-1}(0)=\mathbb{R} \setminus E$ is not F-closed in $\mathbb{R}$.

We note that $f$ is F-continuous function because any an open set $U$ then $f^{-1}(U)$ is F-closed set in $\mathbb{R}$.

**Remark (1-16):**

From theorem (1-11), we get the relation among F-irresolute, F-continuous, sub-F-continuous and continuous function as follows:
Continuous function $\rightarrow$ F-irresolute function $\rightarrow$ F-continuous function $\rightarrow$ sub-F-continuous function.

**Definition (1-17):**

A function $f : (X, \tau) \rightarrow (Y, \tau')$ is said to be pre-continuous function iff for any an open set $U$ in $Y$ then $f^{-1}(U)$ is preopen set in $X$.

**Theorem (1-18):**

A function $f : (X, \tau) \rightarrow (Y, \tau')$ is continuous function iff $f$ is pre-continuous and sub-F-continuous function.

**Proof:**

Suppose that $f$ is pre-continuous and sub-F-continuous function and $B$ is a base for $Y$ such that for any $U \in B$ then $f^{-1}(U)$ is F-closed set. Now let $V \in \tau'$ and $f(x) \in V$.

There is $a \in U \subseteq B$ such that $f(x) \in U \subseteq V$.
Since $f^{-1}(U)$ is pre-open and F-closed set then $f^{-1}(U)$ is an open set, therefore, $f$ is continuous function.

**Proposition (1-19):**

Let $f : (X, \tau) \rightarrow (Y, \tau')$ and $g : (Y, \tau') \rightarrow (Z, \tau'')$ two functions, then
1. If $f$ and $g$ are F-irresolute functions, then $gof$ is F-irresolute function.
2. If $f$ is F-continuous function and $g$ is continuous functions, then $gof$ is F-continuous function.

**Remarks (1-20):**
1. The composition of two F-continuous functions need not be F-continuous function.

2. The composition of a sub- F-continuous function and continuous function need not be sub-F-continuous function.

References:


