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F-CONTINUOUS FUNCTIONS AND SUB-F-CONTINUOUS FUNCTIONS

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Abstract:

In this paper we introduce and study F-closed sets and new types of generalized continuity.

Introduction:

A subset A of a topological space X is said to be F-closed if it is the intersection of an open and closed set . in this paper we introduce three different notions of generalized continuity , namely F- irresoluteness , F-continuity and sub-F- continuity and we discuss some properties of these functions

Definition (1-1):

A subset A of a space (X, τ) is called F-closed if $A=U\cap V$ such that U is open set and V is closed set in X. we denote the collection of all F-closed subsets of X by F(X, τ).

Remarks(1-2) :

A subset A of X is F-closed set iff X-A is the union of an open set and a closed set .

1. Any open (resp. closed) subset of X is F-closed set.
2. The complement of a F-closed subset need not be F-closed set .

Definition (1-3) :

A subset A of a space (X , τ) is said to be preopen set if $A\subseteq$ int(cl A).

Remarks (1-4) :

1. Every open set is preopen set .
2. Every preopen and F-closed set is open set .

Proposition (1-5):

Let A be a subset of a space (X, τ) , then the following statements are equivalent:

1. A is F-closed set .
2. $A=U \cap cl\, A$, U is open set in X .
3. cl A- A is closed set .

Remark (1-6) :

Let A any sub set of a space (X, τ) then A need not be F-closed set , but if (X, τ) has property which every dense subset of X is open set then A is F-closed set .

Proposition (1-7) :

Let A and B be F-closed subsets of a space (X, τ) . If $A \cap clB=clA \cap B=\phi$, then $A\cup B\in F(x, \tau)$.

Proof :

Suppose there are open sets U and V such that $A=U\cap clA$ and $B=V\cap clB$.

Since $A \cap clB=B\cap clA=\phi$, then $A\cup B=(U\cup V) \cap cl(A\cup B)$, from definition of F-closed set we obtain $A\cup B\in F(X, \tau)$.

Definition (1-8) :

A function $f : (X, \tau) \rightarrow (Y, \tau')$ is said to be F-irresolute function iff for any F-closed set U in Y then $f^{-1}(U)$ is F-closed set in X .

Definition (1-9) :

A function $f: (X, \tau) \rightarrow (Y, \tau')$ is said to be F-continuous function iff for any open set U in Y then $f^{-1}(U)$ is F-closed set in X .

Definition (1-10) :

A function $(X , \tau) \rightarrow (Y, \tau')$ is said to be sub-F-continuous function if there is a subbase or base B for Y such that for any $U\in B$ then $f^{-1}(U)$ is F-closed set in X .

Theorem (1-11) :

Let $f :(X, \tau) \to (Y, \tau')$ be a function , then

1. If f is continuous function then f is F-irresolute function.
2. If f is F-irresolute function then f is F-continuous function .
3. If f is F-continuous function then f is sub-F- continuous function .

Remark (1-12) :

The converse of theorem above is not true in general. The following examples explain that.

Example (1-13) :

Let $f : (R , \tau_u) \to (R , \tau_u)$, τ_u is usual topology on R , we will define f on R as follows : $f(x) =1$ if $x > 0$ and $f(x) = \mathbf{x}$ if $x \leq 0$

We note that f is not continuous function but f is F-irresolute function because for any F-closed set U in R then $f^{-1}(U) =U\cup (0,\infty)$ if $1\in U$ and

$f^{-1}(U)=U \cap (-\infty, 0)$ if $1\notin U$, $U\cup (0,\infty)$ and $U \cap (-\infty, 0)$ are F-closed sets ,therefore , f is F-irresolute function .

Example (1-14) :

Let $f : (R , \tau_u) \to (R , \tau_u)$ such that $f(x)=x$ if $x \neq 0$ and $f(0)=1$. For any $U\subset R$ we have $f^{-1}(U)=U-\{0\}$ if $1\notin U$ and $f^{-1}(U)=U\cup\{0\}$ if $1\in U$.

Hence , if U is an open interval then $f^{-1}(U)$ is F-closed . thus f is sub-F-continuous function , but f is not F-continuous function because there is an open set $U=R-\{0\}\cup\{1\backslash n \mid n\in N , n\geq 2 \}$ and $f^{-1}(U)=\{x\in R \mid x \neq 1\backslash n$ for each $n\geq 2\}$ is not F-closed set .

Example(1-15) :

Let $E=\{1\backslash n , n\in N\}$, let $f : (R , \tau_u) \to (R , \tau_u)$ such that $f(x)=x$ if $x\in E$ and $f(x)=0$ if $x\in R-E$, f is not F-irresolute function because {0} is F-closed set in R but $f^{-1}(0)= R-E$ is not F- closed in R

We note that f is F-continuous function because any an open set U then $f^{-1}(U)$ is F-closed set in R.

Remark (1-16) :

From theorem (1-11) , we get the relation among F-irresolute ,F-continuous , sub-F-continuous and continuous function as follows

Continuous function →F-irresolute function →F- continuous function → sub-F-continuous function.

Defintion (1-17) :

A function $f :(X, \tau) \to (Y, \tau')$ is said to be pre-continuous function iff for any an open set U in Y then $f^{-1}(U)$ is preopen set in X .

Theorem (1-18) :

A function $f :(X, \tau) \to (Y, \tau')$ is continuous function iff f is pre-continuous and sub-F-continuous function .

Proof :

Suppose that f is pre-continuous and sub –F-continuous function and B is a base for Y such that for any $U\in B$ then $f^{-1}(U)$ is F-closed set. Now let $V\in \tau'$ and $f(x) \in V$.

There is $a\in U\in B$ such that $f(x) \in U\subseteq V$.

Since $f^{-1}(U)$ is pre-open and F-closed set then $f^{-1}(U)$ is an open set , therefore , f is continuous function .

Proposition (1-19) :

Let $f :(X, \tau) \to (Y, \tau')$ and $g:(Y, \tau') \to (Z, \tau'')$ two functions **,** then

1. If f and g are F-irresolute functions, then gof is F-irresolute function.
2. If f is F-continuous function and g is continuous functions, then gof is F-continuous function .

Remarks (1-20) :

1. The composition of two F-continuous functions need not be F-continuous function .
2. The composition of a sub- F-continuous function and continuous function need not be sub-F-continuous function .

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