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Mθ - Closed Set In T_0 -MAlexandroff Spaces

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الملخص

في هذا البحث، قدمنا تعاريف جديدة للصفوف Mθ – مجموعة مغلقة، Mθ - مجموعة تجمع و Mθ – مجموعة مفتوحة، بالإضافة لإعطائنا مبرهنات وحصلنا على نتائج تتعلق في تلك المجاميع في فضاء ألكساندروف ألأصغري.

Abstract

In this paper, we introduce definitions of Mθ-closed, Mθ-adherent and Mθ-open set, and we give some theorems and results related to these sets in minimal Alexandroff space.

Keywords: Alexandroff space, minimal space, Mθ-closed.

1. Introduction

An Alexandroff spaces which are introduced the first time by (Alexaandroff ,1937). These spaces satisfy the property that an arbitrary intersection of open sets is open. Maki (1950), introduced the notions of minimal structure and minimal space, these spaces satisfy the condition that Φand X belong to a family of subsets of X. Also (Alimohammady ,2005) obtained several new and interesting results related to these spaces. So, we must recall, (Velicko , 1968) introduced θ -open sets, which are stronger than open sets, in order to investigate the characteristics of *H*-closed spaces. Some authors (see Caldas, 2004, Dickman and Porter ,1977 and Dickman and Porter ,1975) studied the subject. The collection of all θ-open sets in a topological space (X, T) forms a topology T_θ on X, weaker than T. Following Velicko, a point x of a space X is called a θ-adherent point of a subset A of X if cl(U)⋂ A ≠ ϕ. For every open set U containing x. The set of all θ-adherent points of A is called the θ-closure of A, and denoted by cl_θ (A). A subset A of a space X is called θ-closed if A = cl_θ(A.) The complement of a θ-closed set is called θ-open. Similarly, the θ-interior of a set A in X , written Int_θ(A), consists of those points x of A such that for some open set U containing x, such that cl(U) ⊆ A . A set A is said to be θ-open if A = Int_θ A, or equivalently, X-A is θ-closed.

In previous paper, we introduced minimal Alexandroff space which are minimal space, and defined the intersection of any family of minimal open sets is minimal open (Hashoosh and Farawi, 2012). Equivalently , every point x in a minimal space X has a minimal open set denoted by $\vec{\Gamma}$x.

2. Preliminaries

Definitions 2.1 (Maki ,1950)

1) Let X be a nonempty set a family $M \subseteq P(X)$ is said to be minimal structure on X if $\emptyset, X \in M$. In this case (X, M) is called a minimal space.
2) A set $A \in P(X)$ is said to be an m-open if $A \in M$, $B \in P(X)$ is a m-closed set if $B^c \in M$. We define the interior and closure of a subset of A, with respect to minimal space

$$m - Int(A) = \cup \{U : U \subseteq A, U \in M\}$$

$$m - Cl(A) = \cap \{F : A \subseteq F, F^C \in M\}.$$

Definitions 2.2 (Maki ,1950)

i) Let (X, M) be an m-space then we say that (X, M) has the property U if the arbitrary union of m-open sets is a m-open set.

ii) Let (X, M) be an m-space then we say that (X, M) has the property I if that any finite intersection of an m-open sets is m-open.

Definition 3. (Alexaandroff ,1937).

An Alexandroff Space is a topological space in which arbitrary intersection of open sets is open.

T_0- Alexandroff spaces are in one-to-one correspondence with posts, via the relation called (Alexandroff) specialization order (≤) , $x \leq y$ if and only if $x \in cl\{y\}$, and each one is completely determined by the other (Alexandroff ,1937).

3. Mθ. closed set in M- T_0. Alexandroff spaces

In this section, we give some important definitions and theorems for minimal Alexandroff Space.

Definition 3.1 (Hashoosh and Farawi, 2012).

Aminimal-Alexandroff space is a minimal space (m-space), in which arbitrary intersection of m-open sets is an m-open, and denoted by: (M^A-space).

Definition 3.2 (Hashoosh and Farawi , 2012).

A minimal-Alexandroff space (X, M^A) is T_0-space if and only if for each distinct points x and y of X there exists an m-open set contains one of the points and not containing the other point, and denoted by (T_0-M^A-Space).

Remark 3.3

Every Alexandroff space is minimal Alexandroff space.

Proof: - It is clear that from definition (2.3) and (3.1).

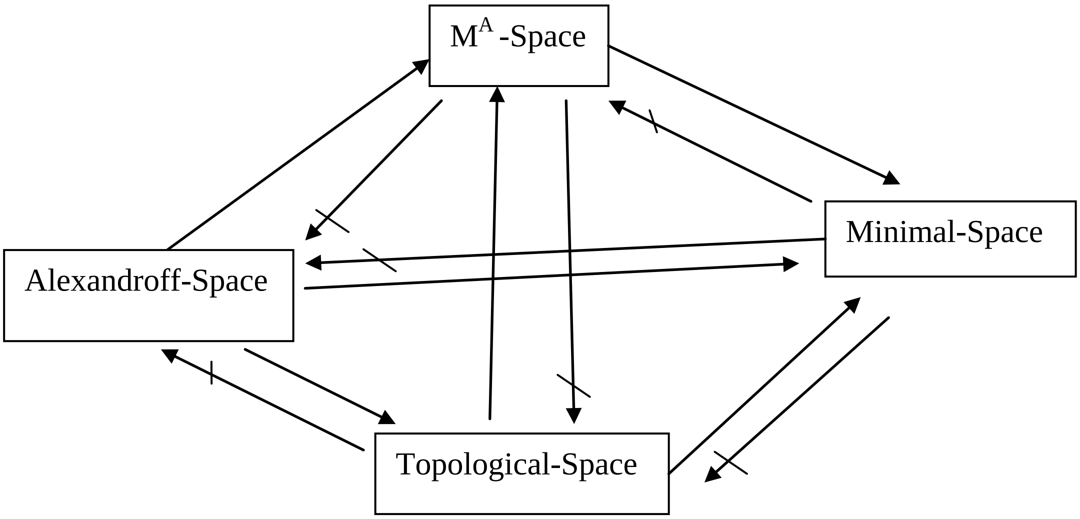
The following example explains the converse of the above Remark is not true in general.

Example 3.4

If $X = \{a, b, c\}$, $M = \{\phi, X, \{a\}, \{a, c\}\}$: $M^A = \{\phi, X, \{a\}, \{c\}\}$.
It is clear (X, M^A) is not Alexandroff space. So, (X, M) is not M^A-space.

Remark 3.5

We have the following diagram:



Definition 3.6

Let $(X, \leq)$ be aT_0-M^A-Space and $A \subseteq X$, $a \in X$. The following notations have the following meanings:

$\uparrow a = \{y \in X : y \geq a\}$.

$\uparrow A = \cup \{\uparrow x : x \in A\}$.

$\downarrow a = \{y \in X : y \leq a\}$.

$\downarrow A = \cup \{\downarrow x : x \in A\}$.

Definition 3.7

Let (X, M) be a minimal space has the property I. A point $x \in X$ is said to be minimal θ-adherent point of $S \subseteq X$ if $(S \cap m\text{-}cl(U)) \neq \emptyset$ for every m-open set U containing x, the set of all minimal θ-adherent point of S is called the minimal θ-adherent of S and is denoted by Mθ-Cl(S).

Definition 3.8

Let (X,M) be an minimal space has the property I , S $\subseteq$ X is called minimal θ-closed if Mθ-Cl(S) = S , and denoted by Mθ-closed .The complement of an Mθ-closed set is called minimal θ- open (Mθ-open).

Definition 3.9

A point x ∈X is said to be an Mθ - interior point of A if there exists an m-open set U containing x such that U $\subseteq$ m-cl(U) $\subseteq$A. The set of all Mθ-interior points of A is said to be the Mθ - interior of A , and it is denoted by M-int$_\theta$(A) . A is Mθ-open iff A = M-Int$_\theta$(A).

Remark 3.10

Let (X, M) be a minimal space has the property I.

(i) If S $\subseteq$ X , then S $\subseteq$m-Cl (S) $\subseteq$Mθ-Cl (S)

(ii) Every Mθ-closed is m-closed.

(iii) Every Mθ-open set is m-open.

Theorem 3.11

If X is aT_0-M^A-Space and A is am-open subset of X, then Mθ.Cl(A)= m-Cl(A).Proof . ⇒m-Cl (A) $\subseteq$Mθ-Cl (A) , it is clear from Remark (3.10) .

Let x ∈Mθ-Cl(A) then for each M-open set U containing x ,then(m-Cl(U) ⋂ A) ≠Ø. So there exists a ∈A such that a∈m-Cl (U). Therefore a∈ ↳ v for some v∈U , and hence v ∈↱a ⋂U, since ↱a $\subseteq$ A, we have A ⋂ U≠ Ø, therefore x∈m-Cl (A).

Theorem 3.12

Let X be a T_0-M^A-Space and let A be a non-empty subset of X then m-Cl (↱ A) = Mθ-Cl (A).

Proof. Suppose that x ∈Mθ-Cl (A) and V be an m-open set containing x then m-Cl(V)⋂A≠Ø .So there exists a ∈m-Cl(V) and a ∈A. Thus a ∈↳ u for some u ∈V, therefore u∈↱ a, hence u∈↱ A ⋂ V ≠ Ø . Hence x ∈m-Cl (↱ A).

Corollary 3.13

Let X be a T_0-M^A-Space, and let A be a non-empty m-open set of X. Then↳x ⊆ Mθ-Cl (A) ,for every x∈Mθ-Cl (A) .

Proof. Because A is an m-open set , then A=↱ A , hence m-Cl (A) = m-Cl (↱ A) =Mθ-Cl (A) is an m-closed set of m-space X , thus ↳x ⊆Mθ-Cl (A).

Corollary 3.14

Let X be a T_0-M^A-Space, and let A be a non-empty set of X , then m-Cl(↱ A) = Mθ-Cl(↱ A).

Proof. Since↱ A ∈ M . So by the theorem (3.12) , m-Cl (↱ A) = Mθ-Cl (↱ A)

Theorem 3.15

Let X be aT_0-M^A-Space, and let A be a nonempty m-open set of X, then↱x⋂Mθ-Cl(A) = Ø ∀ x∉Mθ-Cl(A).

Proof. Suppose to contrary that , there exists x∉Mθ-Cl (A) such that ↱x⋂ Mθ-Cl(A)) ≠ Ø. So , there exists u ∈↱x and u∈Mθ-Cl(A) , since from corollary (3.13) we get x∈↳u ⊆ Mθ-Cl(A)which is a contradiction .

Theorem 3.16

Let X be a T_0-M^A-Space, and let A be a nonempty m-open set of X. Then A is Mθ-open set if and only if A is Mθ-closed.

Proof. A is Mθ-open iff ∀x∈ A,m-Cl(↱x) ⊆A iffm-Cl(↱A) ⊆ A. But A ⊆ m-Cl(↱A) so A is Mθ-open iff m-Cl(↱A) = A = Mθ-Cl(↱ A). From Theorem

(3.12) and definition (3.8), we get A is Mθ-closed.

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